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THE DECAY OF WALL-BOUNDED MHD TURBULENCE AT LOW Rm

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Abstract: We present DNS of decaying Magnetohydrodynamic (MHD) turbulence between two Hartmann walls, at low magnetic Reynolds number and high Hartmann numbers, up to 896. It is found that the decay proceeds through two phases: first, energy and integral lengthscales vary rapidly during a two-dimensionalisation phase extending over about one Hartmann friction time. Once the large scales are close to quasi-two dimensional, the decay results from the competition of a two-dimensional dynamics driven by dissipation in the Hartmann boundary layers and the three-dimensional dynamics of smaller scales. In the later stages of the decay, a purely quasi-two dimensional decay dominated by friction in the Hartmann layers is not reached, because of residual dissipation in the bulk. Also, velocities and transport along the magnetic field are strongly suppressed, in agreement with experiments of (1).

Key words: Magnetohydrodynamics, freely decaying turbulence, turbulence dimensionality, vortex dynamics, two-dimensional turbulence, quasi-two dimensional flows.

1. Introduction When the magnetic field $B\mathbf{e}_z$ is imposed (in the sense of the low magnetic Reynolds number approximation ((2)), turbulence evolves as a result of the competition between inertia and the diffusion of momentum along the direction of the magnetic field. A structure of size $l_\perp$ becomes elongated over a length l_z by this diffusion over a timescale of $\tau_J(l_z/l_\perp)^2$ ((3)), whilst loosing energy through Joule dissipation ($\tau_J = \rho/(\sigma B^2)$ is the Joule dissipation time, ρ and σ are the fluid density and electric conductivity.). (4) first showed that under this linear phenomenology, the turbulent kinetic energy decayed at $E \sim (t/\tau_J)^{-1/2}$ towards an asymptotic state where the flow quantities did not vary along the magnetic field (in this sense, a *two-dimensional* state) but where the kinetic energy of the component along the magnetic field was a third of the total kinetic energy (for a three-component flow).

When Hartmann walls are present, a strictly two-dimensional state is not possible because of the very thin Hartmann boundary layers that develop along them (see for instance (5)). Instead, (3) theorised that in a channel of width L, a structure of size $l_\perp$ became quasi-two-dimensional after $\tau_{2D}(l_\perp) \sim \tau_J(L/l_\perp)^2$. Past this stage, electric current in the core became of order Ha^{-1}, the ratio of the boundary layer thickness to L: dissipation occurs then almost exclusively in the boundary layers where it is equally viscous and magnetic. In contrast, strictly two-dimensional states are possible when walls are absent and the Joule dissipation can therefore drop to much lower values. (1)

also observed experimentally that in the presence of Hartmann walls, transport along the magnetic field was suppressed in the later stages of the decay, in contrast with the findings from numerical simulations in periodic domains (6).
Here, to resolve some of these contradiction, we investigate decaying turbulence in a channel bounded by electrically insulating walls. We focus on the questions questions, which are currently left open:

1. Does three-dimensionality subsist in the later stages of the decay ($t \gg \tau_{2D}(l_\perp)$)?
2. Which part of the energy subsists in the third velocity component ?
3. How do Hartmann walls affect the early phases of the decay ($t < \tau_{2D}(l_\perp)$) ?

2. Governing equations

2.1. Problem definition At low Magnetic Reynolds number, the full system of the induction equation and the Navier-Stokes equations for an incompressible fluid can be approximated to the first order the Magnetic Reynolds number Rm, which represents the ratio of the induced magnetic field to the imposed one. Under this approximation, the Lorentz force expresses as a linear functional of the veloctiy field and the non-dimensional governing equation take the following form:(2):

$$\frac{\partial \mathbf{u}}{\partial t} + P[(\mathbf{u} \cdot \nabla)\mathbf{u}] = \Delta \mathbf{u} - \frac{1}{Ha^2}\Delta^{-1}\partial_{zz}\mathbf{u}\,, \tag{1}$$

where $Ha = LB\sqrt{\sigma/\rho\nu}$ is the Hartman number, P denotes orthogonal projection onto the subspace of solenoidal fields, $\mathbf{u}$ denotes the fluid velocity, $\mathbf{B}$ the externally imposed magnetic field, ν the kinematic viscosity, and σ the electrical conductivity.
The geometry of the problem is that of a channel flow with a homogeneous transverse magnetic field $B\mathbf{e}_z$ and impermeable ($\mathbf{u}|_{wall} = \mathbf{0}$), electrically insulating ($\mathbf{j} \cdot \mathbf{n}|_{wall} = \mathbf{0}$) walls located at $z = \pm L/2$. Periodic boundary conditions of period L are imposed in the x and y directions.

2.2. Numerical method The problem is solved numerically, using a new type of spectral method designed to alleviate the computational cost associated with strong anisotropy and thin Hartmann boundary layers. The main idea is to take advantage of the fact that such structures are dissipative and that their presence should reduce the number of degrees of freedom of the flow, when paradoxically, their fine resolution incurs extra computational cost in most current methods. The mathematical principle is to use a functional basis with elements that already include these fine structures so as to avoid these extra costs. The main advantages of this technique are that Hartmann layers are finely resolved with a limited number of collocation points so in contrast to traditional methods, the computational cost per time step does not depend on the intensity of the magnetic field. This makes it possible to simulate flows at much higher magnetic fields than standard methods (such as spectral methods based on Techbychev polynomials). Another advantage of this technique is that it can potentially be adapted to a number of other problems where fine resolution becomes problematic in extreme regimes, as long as a suitable basis can be found. The mathematical foundations of this method and its numerical implementation are described in detail in (7; 8), where it is also tested for the exact channel geometry studied here.

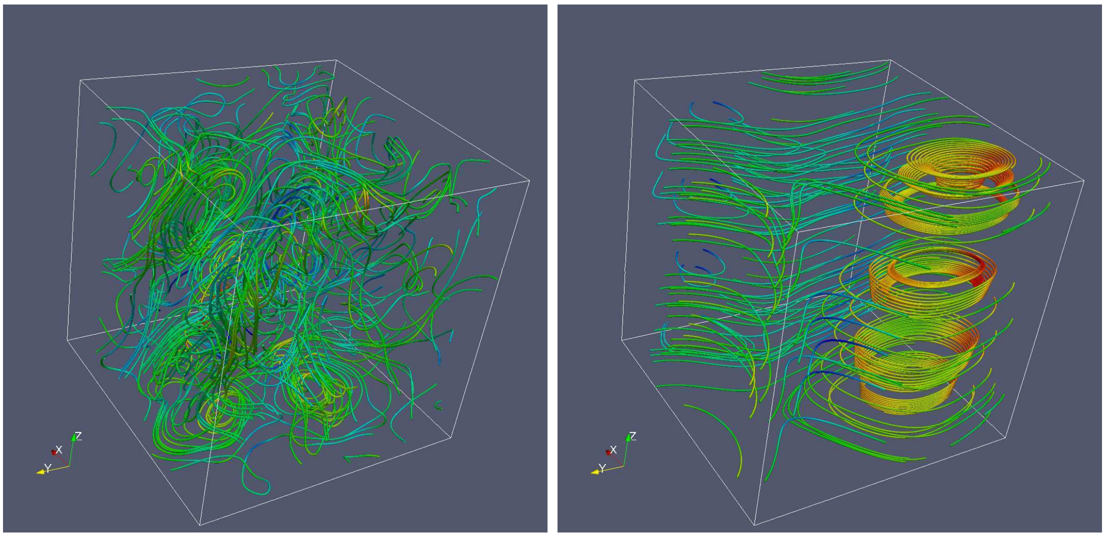

Figure 1: Instantaneous streamlines for $Ha = 112$ at $t = 0$ & $t = 100\tau_J$

We use three types of boundary and initial conditions: first, as in (9), the initial conditions consist of a isotropic, random Gaussian velocity field with $u(k) \sim \exp\left[(-k/k_p)^2\right]$ where $k_p = 4\pi/L$. To single out the influence of Hartmann walls, these initial conditions are either used in conjunction with a 3D periodic domain or adapted to the geometry described in section 2.1 (referred to as "walls 3D IC" and "periodic" thereafter). A third type of configuration consist of the same geometry with Hartmann walls, but with an initial flow field that is isotropic in the $x - y$ plane and constant along z, except near Hartmann walls, where Hartmann layers exist ("walls 2D IC"). The Reynolds number $Re = u'l_0/\nu$ is based on the initial integral lengthscale l_0 and velocity $u' = u(k = k_p)$ and is initially set to 336. The Hartmann number spans values between 112 and 896.

3. Results The decay exhibits three- and a two-dimensional phases with an overlap (Snapshots from the initial flow and the second phase are shown on figure 1): the former is dominated by the two-dimensionalisation process, where diffusion by the Lorentz force stretches vortices until they reach the Hartmann walls. This process is highly dissipative and leads to a rapid variation of energy and of the integral lengthscale along **B** (see Fig. 2 (a) and (b)). Larger scales are two-dimensionalised more quickly than smaller ones. Once the large scales of turbulence are close to two-dimensional (after approximately $\tau_{2D}(l_0)$) the flow starts exhibiting a two-dimensional dynamics, where dissipation mostly takes place in the Hartmann boundary layers, with a slower characteristic timescale $\tau_H = Ha\tau_J$ (Fig. 2 (e) and (f)). However since it can take up to τ_H for small scales to adopt a two-dimensional dynamics, there is no clear separation between these two phases and both two and three-dimensional dissipation mechanisms co-exist long after $\tau_{2D}(l_0)$. Several important features of this phenomenology stand out:

First, the presence of the walls turns out to impede the growth of l_z right from the earliest stages of the decay, whereas the decay of energy remained roughly in line with (9)'s law of $E \sim t^{-1/2}$ for unbounded turbulence in the limit of high Ha, during around one Joule time (Fig. 2 (a) and (b)).

Second, the energy associated to the velocity component across the channel is very strongly suppressed: E_z/E tends to 0 much faster than for unbounded turbulence (Fig. 2 (c)). This is in line with (1)'s experiments and confirms that walls are responsible for the suppression of the third component. Further evidence of this suppression is visible in the long

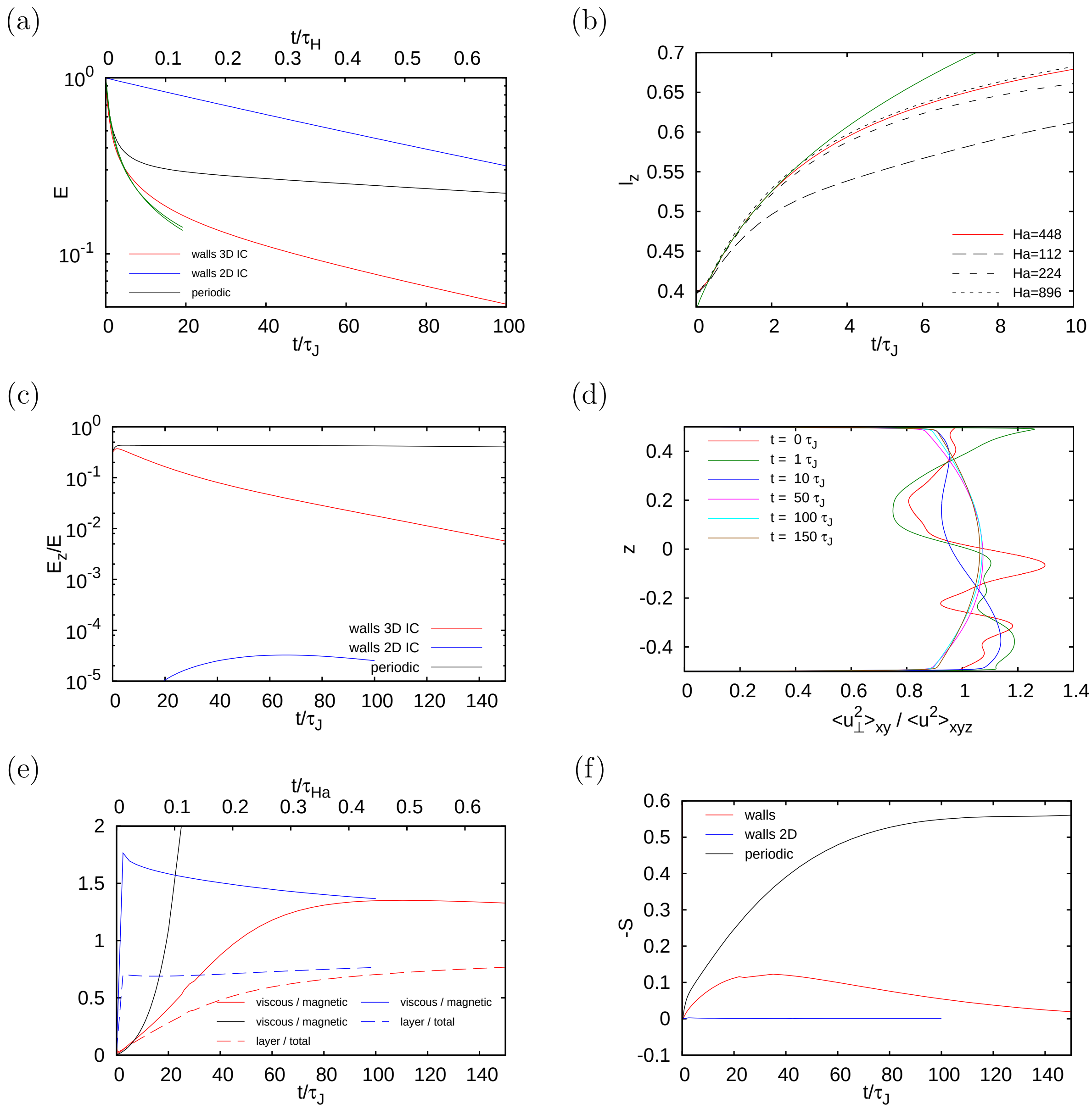

Figure 2: (a) Evolution of normalised kinetic energy, green line: best fit to a law of the form $K(1+\beta\frac{t}{\tau_J})^{-0.53}$ put forward by (9). (b) integral lengthscale, green line, best fit to $K(1+\beta\frac{t}{\tau_J})^{0.26}$, which is similar to ((9)) prediction but with a lower exponent. (c) ratio of kinetic energy in the z-velocity component to total kinetic energy for $Ha = 448$. (d) normalised instantaneous profiles of velocities $\langle\rangle_{xy}$ and $\langle\rangle_{xyz}$ respectively denote spatial averaging along x and y directions only and in the entire domain. (e) ratios of total viscous to total magnetic dissipation and of dissipations in the Hartmann layer to dissipation in the bulk. (f) Skewness.

term behaviour of the skewness which tends to 0 in the case with walls. With periodic boundary conditions, by contrast, the skewness remains high and almost constant during the entire two-dimensionalisation phase, of order τ_H (Fig 2 (f)). Thirdly, long into the "two-dimensional phase", even at the highest value of Ha, a form of three-dimensionality subsists (Fig. 2 (d)), due to currents recirculating between the Hartmann layers and the bulk. This effect is characterised by the barrel shape visible on the larger structures, as predicted by (10). Though less pronounced at higher values of Ha, our simulations show no evidence of it vanishing at larger times.

Fourthly, in quasi-two dimensional flows dominated by dissipation in the Hartmann boundary layers, the total kinetic energy would be expected to decay exponentially with a timescale of τ_H. However, a true exponential decay of this sort was never observed, even for $t > \tau_H$. Remarkably this discrepancy to a pure exponential decay did not result from the residual three-dimensionality due to the barrel effect, but mostly from viscous friction in the horizontal plane. Greater detail of the mechanisms of the decay can be found in (11).

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