TOPOLOGICAL CONSTRAINTS IN QUASI-2D FLOWS AND MODELS OF PLANETARY CORES

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In liquid planetary cores, the rate at which the planet cools, spins-down and the dynamics of its magnetic field are all controlled by the complex interplay between buoyancy, the Coriolis force due to planetary rotation and the Lorentz force. Yet the combination of these three forces and the extreme regimes in which they operate makes the resulting rotating turbulent magneto-convection particularly arduous to elucidate. The main effect of rotation is to oppose fluid motion across an imaginary surface in the shape of a so-called *Tangent Cylinder* (TC) extruded from the equatorial perimeter of the solid inner core along the rotation direction, and up to the boundary between the liquid core and the mantle [Aurnou *et al.*, Earth Planet. Sci. Lett. **212**, 119 (2003)]. Magnetic fields on their own have a similar effect, that is well documented in liquid metal flows [Alboussière *et al.*, **8** 2215-2226, Phys. Fluids (1996)].

As it turns out, a much wider class of flows exhibits a similar behaviour to rotating and MHD flows. In fact, any flow subjected to a dominating planar force exhibits a tendency to two-dimensionality, with the consequence that certain flow quantities are channelled along Characteristic surfaces determined by the shape of the fluid domain, the properties of its boundaries and the inhomogeneity of background fields such as Rotation or Magnetic fields. Geostrophic contours in rotating flows belong to this class of surfaces, so do the Characteristic surfaces in Quasi-static MHD.

We apply this formalism to magneto-rotating convection in a geometry relevant to planetary cores in the limit of Quasi-Static MHD. In doing so, we show that the classical Taylor-Proudman theory can be extended into a new constraint that applies to the combined current density of both mass and charge. In the case of magneto-rotating convection, this constraint translates into a kinematic equation linking the radial and the azimuthal components of the flow at the TC boundaries. This adds to the previous suggestion that a radial flow may exist across the Earth's TC [Cao *et Al.* **115** (44) 11186-11191, PNAS (2018)]
The theory is tested on the *Little Earth Experiment*(LEE [Aujogue *et al.*, Rev. Sci. Instrum. **87**,084502 (2016), Aujogue *et al.*, J. Fluid Mech. **843**, 355

(2018)]), an original device where rotation, magnetic field and buoyancy can be controlled, and where rotating magneto-convective patterns are visualised for the first time. The principle of the experiment is to model the liquid core with a vessel representing the core-mantle boundary, with a cylindrical heating element placed at its centre modelling the solid inner core and the buoyancy it creates. The vessel is filled with a transparent electrolyte, driven in rotation and placed inside a large magnet imitating the feedback of the Earth's magnetic field on the flow. Particle Image Velocimetry and thermistors provide us with velocity maps and local temperature measurements. We operate LEE in regimes where the flow inside the TC is either 3D or quasi-2D, and show that the time- and azimuthally- averaged radial flow near the TC boundary indeed follows the prediction of the Magnetic Taylor-Proudman constraint.