INVESTIGATION OF ACOUSTIC STREAMING JETS IN LIQUID

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Abstract: The present paper performed a theoretical study of acoustic streaming. The approach is based on the time scale separation method where the streaming jet is considered varying very slowly in time compared to the acoustic period. An acoustical force is introduced in the Navier-Stokes equations and ensures the coupling between sound propagation and hydrodynamics. Dimensional analysis is used to give first clues for the theory of physical modeling. Through scaling analysis, two scaling laws featuring linear and square root variations of the streaming velocity level with the acoustic power have been found.

1. Introduction

"Not only motion can create sound but also sound can create motion" [1]. This sentence by Sir J. Lighthill explains in a few words what acoustic streaming is: the possibility of driving stationary and quasi-stationary flows using acoustic waves. This phenomenon can be present in many applications ranging from biomedical applications (low intensity ultrasounds based diagnostics and high intensity ultrasounds based treatment) to engineering applications (sonochemisty, velocimetry and potentially crystal growth).

Acoustic streaming is often presented as a second order flow with respect to acoustic as explained by Nyborg [2] on the basis of a small perturbation approach. However, this method seems to be limited because of two reasons. Firstly, assuming streaming velocity of second order leads to a motion equation featuring an acoustical force, pressure gradient and viscosity terms but negligible inertial terms since these are of fourth order. Secondly, in several experimental observations [3, 5], acoustic streaming velocities is not of second order with respect to the acoustic velocity wave propagation. Lighthill [1] suggested thus that this development is suitable for creeping motion which he characterizes with a Reynolds number less than one; for the other Reynolds number, he proposed to introduce inertial terms artificially.

Here, a new approach is proposed. First, we developed a scale separation method as an alternative to Nyborg's approach. Then, we focused to a dimensional analysis in order to consider an application to liquid metals. Finally, a scaling analysis of acoustic streaming jet provides flow velocity evolution as a function of the governing parameters of the problem.

2. Time scale separation method

Each variable (velocity, density and pressure) of the acoustic streaming problem is split into an acoustic part, varying rapidly, and a streaming motion part, varying very slowly compared to the acoustic part: it's the time scale separation method. Introducing these variables into the incompressible Navier-Stokes equation and averaging over an acoustic period, like RANS computation, we find:

$$\begin{cases} div\vec{u}_e = 0 \\ \rho\dfrac{d\vec{u}_e}{dt} = -gradp_e + \mu\Delta\vec{u}_e + \vec{f}_{ac}\ , \\ f_{ac,i} = -\rho div\left(\overline{u_{ac,\iota}\vec{u}_{ac}}\right) \end{cases} \tag{1}$$

where $\vec{u}_e$ represents the flow velocity, $\vec{u}_{ac}$, the acoustic wave velocity, p_e, the hydrodynamic pressure and ρ the fluid density. The additional volumetric force term $\vec{f}_{ac}$ in the incompressible Navier-Stokes equations ensures a coupling between the acoustic propagation and the hydrodynamic flow. This acoustic streaming force must be computed from the spatial variations in acoustic field, which are an output of the acoustic propagation problem. A commonly accepted expression, valid under the plane wave approximation, is:

$$\vec{f}_{ac} = \frac{2\alpha}{c} I_{ac}\vec{x}, \tag{2}$$

where I_{ac} is the acoustic intensity. Through this expression is not new [2], the time scale separation approach allows it to appear naturally in the full incompressible Navier-Stokes equations, which is consistent with experimental observations [3, 5]

3. Dimensional analysis

Since the pioneering work done by Eckart [6], a classical modeling approach is to consider acoustic streaming as the weak coupling between two sub-problems: acoustic propagation and the hydrodynamic flow. The first sub-problem, the acoustic propagation, consists in the description of the acoustic beam; in the framework of linear acoustics, the inputs for this problem are the ultrasounds source diameter d_s, frequency f, and power P_{ac}, and the liquid acoustic properties, *i.e.* sound celerity c, and acoustic attenuation coefficient α. The hydrodynamics sub-problem consists in the description of the quasi-steady flow driven by acoustic streaming; the inputs for this problem are the geometry of the fluid domain and the liquid mechanical properties, namely its kinematic viscosity ν, and density ρ. Ten dimensional variables are thus used; they are listed in table 1 with their corresponding usual units. The Vashy-Buckingham theorem implies that a set of seven dimensionless groups are necessary to describe the whole problem of acoustic streaming. We chose to define d_s, ν/d_s^2 and ρ/d_s^3 as characteristic distance, time and weight, respectively; this leads to the dimensionless groups listed in the last column of table 1.

Dimensional variable	Usual units	Fundamental units	Corresponding dimensionless groups
$N = \alpha/f^2$	$m^{-1}.Hz^{-2}$	$m^{-1}.s^2$	$\boldsymbol{N} = Nf^2L$
f	Hz	s^{-1}	$\boldsymbol{F} = fd_s^2/\nu$
$\lambda = c/f$	m	m	$\boldsymbol{S} = 1.22\lambda/d_s$
L, l et d_s	m	m	$\boldsymbol{L} = L/d_s$, $\boldsymbol{l} = l/d_s$
P_{ac}	W	$kg.m^2.s^{-3}$	$\boldsymbol{P} = P_{ac}d_s/(\rho\nu^3)$
U	$m.s^{-1}$	$m.s^{-1}$	$\boldsymbol{U} = Ud_s/\nu$
ν	$m^2.s^{-1}$	$m^2.s^{-1}$	-
ρ	$kg.m^{-3}$	$kg.m^{-3}$	-

Table 1: Variables of the acoustic streaming problem, their units and the corresponding dimensionless groups.

Each dimensionless parameter can be associated to a physical interpretation: $\boldsymbol{N}$ is a ratio between the length of the domain L and the typical attenuation distance $1/\alpha$, $\boldsymbol{F}$ is the dimensionless frequency and can be seen as a ratio of the period and the characteristic time for viscous diffusion of momentum at the scale d_s, $\boldsymbol{S}$ is typically the half angle of the diffraction cone of the sound beam, $\boldsymbol{L}$ and $\boldsymbol{l}$ are simply the ratios of the cavity length and width to the source diameter, $\boldsymbol{P}$ is the injected acoustic power normalized by a typical power dissipated by viscous effects and $\boldsymbol{U}$, the dimensionless velocity, is a local Reynolds number based on the observed velocity and the source diameter.

4. Toward the case of liquid metal

One of the difficulties when dealing with acoustic streaming in liquid metals is that the acoustic attenuation coefficient is not a very well-known property for this type of liquids. The acoustic attenuation coefficient in a liquid, α, is very often assumed to have three contributions. A first contribution is connected with the dynamical (or shear) viscosity μ, a second contribution is related to the bulk viscosity η, and a final contribution takes into account thermal effects. The expression proposed by Nash *et al.* [7] is:

$$N = \frac{\alpha}{f^2} = \frac{2\pi^2}{\rho c^3}\left(\frac{4}{3}\mu + \eta + \frac{c^2\beta^2\lambda T}{C_p^2}\right) \tag{3}$$

where f is the frequency, ρ is the density, c is the wave velocity, β is the thermal expansion coefficient, λ is the thermal conductivity, C_p is the specific heat, and T is the absolute temperature. The dynamical viscosity μ and the properties involved in the thermal contribution can generally be obtained for standard liquids with an acceptable accuracy, so that the main difficulty will come from the estimation of the bulk viscosity η.
We rely on this estimate of the attenuation coefficient, on the developed dimensional analysis and physical modeling techniques to assess the intensity of acoustic streaming expected in a liquid metal experiment. In particular, we consider the similarity of a hypothetic liquid metal set-up with our existing set-up [8]. We find that, under some assumptions, the similarity condition imposes the scale Σ, the ratio in frequency f, attenuation factor N and acoustic power P between the water-test and the liquid metal experiment. Under this condition the ratio in velocity observed in these apparatus is also given. Focusing on the case of liquid silicon and liquid sodium, featuring respectively a very high and a very low melting temperature, the similarity conditions is given in table 2.

	Scale Σ	f_{test}/f_{real}	$\boldsymbol{N}_{test}/\boldsymbol{N}_{real}$	$P_{ac,\ test}/P_{ac\ ,\ real}$	U_{test}/U_{real}
Silicon (1 750 K)	8.2	0.046	0.17	8.9	0.38
Sodium (393 K)	2.5	0.23	0.28	4.9	0.59

Table 2: Similarity conditions for a model experiment in water (subscript test) and a liquid metal experiment (subscript real). The case of silicon is considered in the first line, that of sodium in the second line.

Considering our set up as test experiment, the second line of table 2 shows that, in liquid sodium, a plane transducer of diameter 12 mm operating at 8.6 MHz would induce velocities on the order of 1.7 cm/s with an acoustic power of only 200 mW. Because of the scale Σ, we can also say that these velocities would be obtained after a smaller distance from the acoustic source. As mentioned earlier, this numerical application makes us think that it should be taken care of acoustic streaming side-effects when measuring small velocities by in ADV in liquid metals.

5. Scale analysis

We proposed in a recent paper [8] two scaling laws for acoustic streaming free jets, *i.e.* steady, laminar, acoustic streaming jets in a semi-infinite medium. As no confinement is considered and there is no reason for the jet to feature any significant curvature, the pressure gradient can safely be assumed not to play any significant role. The flow is thus governed by a balance between the combined effects of viscosity, inertia and the acoustic streaming force. We focus successively on the two asymptotic cases of negligible viscous effects and negligible inertia effects. Let us first consider the acceleration zone near the origin of the jet. We assume this region to be dominated by inertia effects, which balance the acoustic streaming force. the typical velocity u_e at a distance (x-x_0) from the origin of the jet is then expected to follow the following scaling law:

$$u_e \approx \kappa_1 \sqrt{\frac{2\alpha}{\rho c} \frac{P_{ac}}{\pi R_{ac}^2} (x - x_0)} \tag{4}$$

where κ_1 is a multiplicative factor of the order of 1.

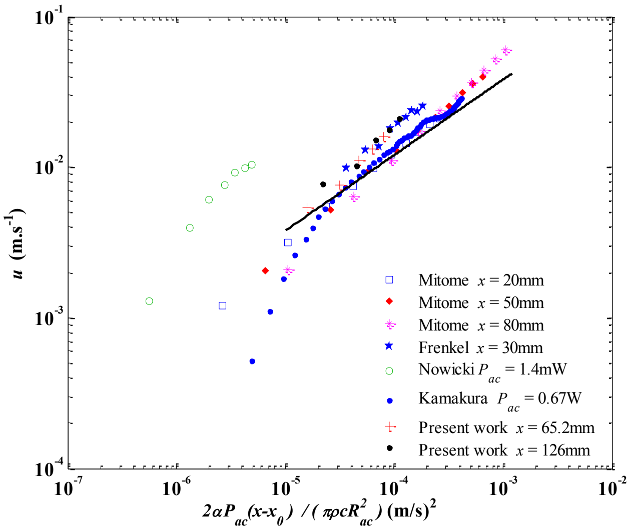
Farther from the origin, we consider that the flow is nearly one-dimensional and ruled by the balance between the acoustic force and the viscous forces;

$$u_e \approx \kappa_2 \frac{\alpha P_{ac}}{\pi \mu c} \tag{5}$$

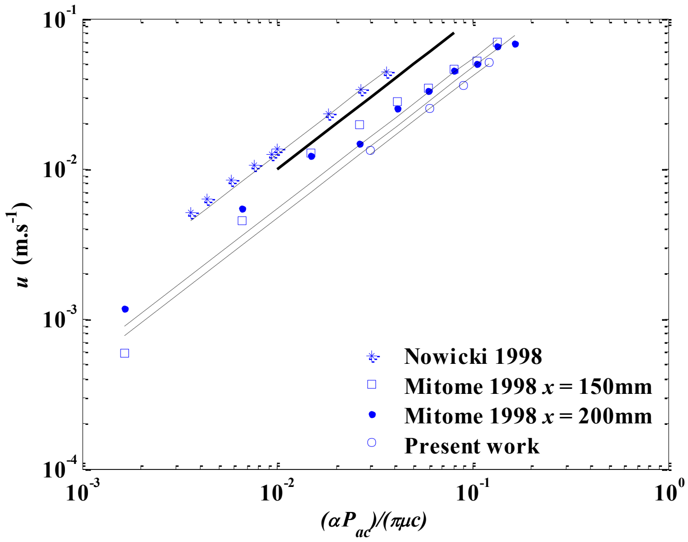
where κ_2 is a multiplicative factor of the order of 1. These two scaling are plotted in figure 1 with experimental data of Mitome [4], Frenkel *et al.* [5], Nowicki *et al.* [3] and Kamakura *et al.* [9] and for the present study. We see that the set of experimental data is in reasonable agreement with the proposed scaling laws and that both scaling laws are observed.

6. Conclusion

Acoustic streaming is a coupling between an acoustic propagation and an incompressible flow ensured by an acoustical force term (eq. (2)) in the Navier-Stokes equation. A time scale separation method was developed to derive this force expression. It consists in separating the short time scale of the acoustic propagation from the long time scale of the hydrodynamic flow. The same acoustical force expression than Nyborg was obtained, but our approach is more consistent with experimental observations: flow velocity is not of second order with respect to the acoustic velocity. Moreover, equations of motions thus naturally feature inertial terms. Ten dimensional variables are present in the acoustic streaming problem. Following this, seven dimensionless parameters are proposed to describe the whole acoustic streaming problem and make similarities with experimental set up with other liquid. In particular, it was found that flow velocity reached in Silicon and Sodium are respectively 2.5 and 1.5 times higher with an acoustic power 9 and 5 times lower. Finally, two velocity scaling laws are obtained for the streaming flow and plots of experimental measurements of the present work and former studies show the reliability of the scaling analysis in the range of parameters.



(a)



(b)

Figure 1: Comparison of the former and present experimental results with the scaling law given by (a) equation (4) and (b) equation (5).

7. References

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