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STABILITY OF MIXED BAROCLINIC CONVECTION IN A NEARLY SEMI-CYLINDRICAL CAVITY

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Summary In this work, we are seeking the critical conditions for the oscillatory instability that arises in a nearly semi-cylindrical cavity fed in with hot fluid at the upper boundary, bounded by a cold, porous semi-circular boundary at the bottom, and infinitely extended in the third direction. In this geometry, the flow is driven by an unusual type of mixed convection, which is the combination of the buoyancy indirectly caused by the shape of the boundaries and the through-flow. Linear stability analysis and direct numerical simulations are performed, using the spectral element method to identify observable states. Our analysis reveals that the unstable modes are three-dimensional. They show oscillatory or non-oscillatory behaviour depending upon the strength of through-flow. The nature of the bifurcation is determined through Stuart–Landau analysis.

We study the convective patterns that arise in cavities with curved isothermal boundaries and permeated by a through-flow (see Fig. 1(a)). This generic configuration is representative of numerous problems where solid materials are melted, for instance, metallurgical casting processes [1]. In this configuration, despite the stable stratification, the pressure gradient and the temperature gradient form an angle, which is maximum near the upper corners. This results in a buoyancy force that cannot be opposed by pressure and leads to a baroclinic imbalance, which drives the base flow [2].

The non-dimensional equations governing stratified flows under Boussinesq approximation are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = RaPrT\mathbf{e}_y + Pr\nabla^2\mathbf{u}, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \nabla^2 T, \tag{2}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{3}$$

where $\mathbf{u}$ is the velocity field, p the pressure field, t is the time, and T is the temperature field. The flow is characterised by three non-dimensional parameters, the Rayleigh number Ra (which is the ratio of buoyancy force to dissipative force), the Prandtl number Pr (which is the ratio of kinematic viscosity to thermal diffusivity), and the Reynolds number Re based on the dimensional feeding velocity u_0. In all simulations, we fixed Pr to 0.02, a value typical of liquid metals in continuous casting processes. Equations (1)-(3) are solved using the spectral element method. We impose a free-slip and a no-slip boundary conditions for the velocity field at the upper free surface and at the front wall respectively. For the temperature field, we impose a conducting boundary condition at both the upper free surface and at the front wall. We apply a no-slip boundary condition for the velocity field and an insulating boundary condition for the temperature field at the right and left side- walls. To satisfy the uniform inflow and outflow, we set vertical velocity to $-u_0$ at the upper free surface and the front wall.

We perform three different types of numerical computations using the open source-code NEKTAR++ [3]. First, we obtain a two-dimensional steady base flow using direct numerical simulation (DNS). Second, we perform linear stability analysis (LSA) of three-dimensional perturbations on the two-dimensional base flow by solving the linearised governing equations (see [2] for details). Third, three-dimensional (3D) DNS are performed in weakly sub- and supercritical regimes

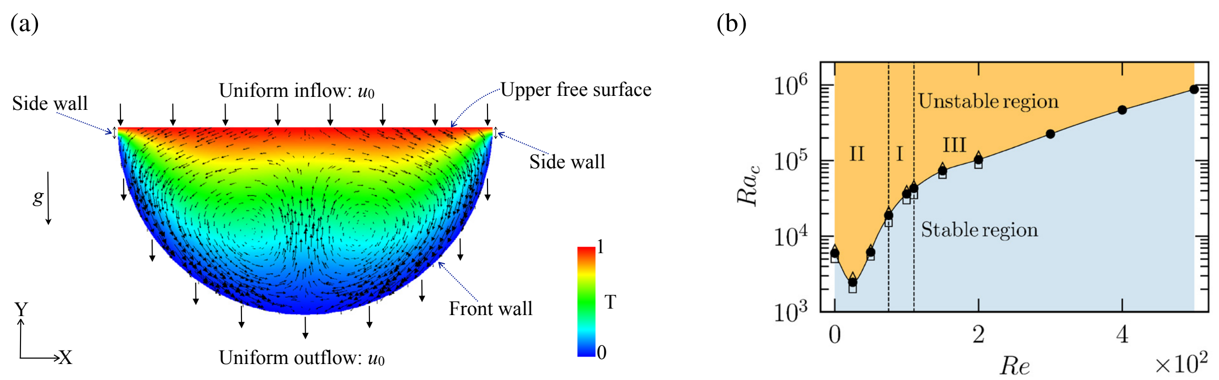

Figure 1: (a) Two-dimensional base flow at $Pr = 0.02$. $Ra = 10^4$, and $Re = 50$. Colours represent the temperature field and arrows represent the velocity vector $\mathbf{u}$. (b) Critical Rayleigh number Ra_c as a function of Re. Blue (orange) regions below (above) the curve represents flow regimes that are linearly stable (unstable) to two or three-dimensional perturbations.

(a) LSA (b) DNS

Figure 2: For $Re = 0$ and $Ra = 7 \times 10^3$: (a) Vorticity perturbation along the homogeneous direction. Out-of-plane vorticity component ζ_z' computed from the linear stability analysis (LSA) at $z = 0$ plane for $k = 6$; (b) ζ_z' computed from the direct numerical simulation (DNS) in the $z = 0$ plane.

to assess the relevance of the linear stability results and to find the nature of the bifurcation by computing the parameters of the Stuart–Landau model [4].

The two-dimensional (2D) base flow is driven by baroclinic imbalance and produces two counter-rotating rolls (see Fig. 1(a) for example). This 2D base flow becomes linearly unstable to three different types of mode, depending on Re at a sufficiently high Rayleigh number. Fig. 1(b) displays the estimate of the critical Rayleigh number Ra_c, at which this transition occurs for a stable state to an unstable state, as a function of Re. When Re is increased from 0, the critical Rayleigh number $Ra_c(Re)$ initially decreases up to $Re = 25$ and then increases. Thus the through-flow has a stabilising influence as soon as the Reynolds number Re exceeds 25. Figure 2 shows the vorticity distributions in a weakly supercritical case for $Re = 0$ and $Ra = 7 \times 10^3$. The snapshots of the topologies of the perturbation from LSA and DNS are presented Fig. 2(a) and Fig. 2(b) respectively, and they match very well. We also find an excellent agreement in the estimate of the growthrate, frequency and the dominant wavenumber between LSA and DNS. Thus the 3D DNS confirm the relevance of the linear stability analysis.

In this study, we observe that the base flow is susceptible to three distinct types of infinitesimal perturbations. For $Re \leq 75$ the most unstable mode corresponds to Branch II as obtained from the linear stability analysis. The three-dimensional DNS show that the most unstable mode is a wave travelling in the $\mathbf{e}_z$ direction. DNS for $100 \leq Re \leq 110$ shows that the instability sets in as a standing oscillation (type I mode), while for $Re \leq 150$ the most unstable mode is non-oscillatory and corresponds to Branch III. Further, from the 3D DNS data, we determined the nature of the bifurcations points through Stuart–Landau analysis for completeness. DNS data revealed that the nature of the bifurcation associated to the three modes varies too. Modes corresponding the branch II and III shows *supercritical* bifurcation, whereas the onset of mode I is *subcritical*.

References

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