Inverse and direct energy cascades in 3D magnetohydrodynamic (MHD) turbulence at low Rm

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Turbulence displays radically opposite dynamics, whether it is three-dimensional (3D) or two-dimensional (2D). In the former, kinetic energy follows a direct energy cascade from the forcing scale down to the smallest scales, while the latter features an inverse energy cascade from the forcing scale up to large structures of the size of the system. It is, however, still unclear how these seemingly irreconcilable dynamics relate to each other, whenever 2D and 3D turbulent structures coexist. This question is all the more crucial when dealing with real-life wall-bounded flows, as speaking of two-dimensionality only makes sense with respect to the presence of boundaries, such as no-slip walls. Yet, solid boundaries necessarily introduce three-dimensionality both in boundary layers and in the bulk. The key question that determines both transport and dissipative properties of such flows is then: how do the energy transfers relate to the topological dimensionality of the individual turbulent structures composing the flow? The present question is tackled experimentally using the Flowcube platform[1,2], which consists of a closed parallelepipedic vessel with an inner square base of width 150 mm, and a height $h = 100$ mm. The vessel is filled with a eutectic alloy of Ga, In and Sn, which is set in motion by forcing a DC current through the bottom wall, while simultaneously applying a static and uniform magnetic field B_0.

We start by identifying a "critical" lengthscale $\hat{l}^c_\perp$ that separates smaller 3D structures from larger quasi-2D ones. This identification is made possible by comparing the scale-wise energy distribution evaluated along the bottom wall (i.e. where the forcing takes place) to that computed along the top wall (i.e. at a distance h away from the forcing), for increasing values of the magnetic field. Plotting the experimentally determined $\hat{l}^c_\perp$ against the true interaction parameter N_t (the latter quantifying the competition between inertia and the Lorentz force's solenoidal component[3]) clearly displays a $N_t^{-1/3}$ region, which provides an experimental confirmation of the theoretical scaling law put forward by Sommeria and Moreau[4].

We then show, by computing the divergence of the third order structure function, that an inverse energy cascade of horizontal kinetic energy along horizontal scales is always observable at large scales, regardless of the turbulent structures supporting this inverse cascade being kinematically 3D or quasi-2D. At the same time, a direct energy cascade confined to the smallest and strongly 3D scales is observed. The dynamics therefore appear not to be simply determined by the dimensionality of individual scales, nor by the forcing scale, unlike in other studies. In fact, our findings suggest that the relationship between kinematics and dynamics is not universal and may strongly depend on the forcing and dissipating mechanisms at play.

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[4]Sommeria and Moreau *J. Fluid Mech.* **118**, 507 (1982).