THE DYNAMICS OF PARTLY 3D / PARTLY 2D TURBULENCE: AN EXPERIMENTAL INVESTIGATION IN THE LOW-RM MHD FRAMEWORK.

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Turbulence displays radically opposite dynamics whether it is three-dimensional (3D) or two-dimensionnal (2D). Indeed, the former features a direct cascade of energy from the injection scale down to the small dissipative scales controlled by viscosity, while the latter features an inverse cascade of energy from the injection scale up to the large structures controlled by the geometry of the system. The goal of this project is to study the dynamics of turbulent flows, which simultaneously feature 2D and 3D turbulent structures. The question is tackled experimentally within the low-Rm magnetohydrodynamic (MHD) framework. Use was made of the Flowcube experimental platform [1], which consists of a closed parallelepiped vessel with an inner square base of width $150\,\mathrm{mm}$, and a height $h = 100\,\mathrm{mm}$. The vessel is filled with Galinstan (a metal liquid at room temperature), which is set in motion by forcing a DC current through the bottom wall, while simultaneously applying a static and uniform magnetic field B_0.

One of the main features of low-Rm MHD is the diffusion of momentum by the solenoidal component of the Lorentz force along the magnetic field [3]. The end result of this diffusive process being the correlation (i.e. the two-dimensionalization) of the flow over the diffusion length l_z^u. Considering a turbulent structure of width $l_\perp$ and velocity $u'(l_\perp)$ laying in the inertial range of fully developed MHD turbulence, the processes at play are: On the one hand the diffusion of momentum by the Lorentz force characterized by the timescale $\tau_{2D} = (\rho/\sigma B_0^2)(l_z^u/l_\perp)^2$; On the other hand, energy transfers characterized by the eddy turnover time $\tau_u = l_\perp/u'(l_\perp)$. In the above, ρ and σ respectively represent the density and electric conductivity of the fluid. In the inertial range, both processes compete with each other, hence introducing the following estimate for the diffusion length $l_z^u(l_\perp)$:

$$\frac{l_z^u(l_\perp)}{h} = \sqrt{N}\,\frac{l_\perp}{h}, \tag{1}$$

where $N = \sigma B_0^2\, l_\perp/\rho\, u'(l_\perp)$ is the local (in scale space) interaction parameter based on the width of the structure in question and its velocity. Equation (1) gives a succinct way of characterizing the dimensionality of the structure by comparing its diffusion length to the height of the channel [2]. Namely, $l_z^u(l_\perp)/h \ll 1$ implies that the turbulent structure of width $l_\perp$ is topologically 3D, as the Lorentz force is not quick enough to diffuse its momentum across the channel before the structure yields its energy to the cascade. Conversely, $l_z^u/h \gg 1$ means that the turbulent structure of width $l_\perp$ is quasi-2D, as the inertial transfers take place over a much longer time scale than that required for the Lorentz force to diffuse its momentum across the experiment. The global dimensionality of the flow may then be estimated experimentally by associating it to the diffusion length of the injection scale, whose size l_i is imposed by the forcing mechanism.

The distribution of turbulent kinetic energy is investigated in scale space by means of the second order structure function $\langle \delta \mathbf{u'_\perp}^2 \rangle(\mathbf{r})$, where $\delta \mathbf{u}'_\perp = \mathbf{u}'_\perp(\mathbf{x}+\mathbf{r}) - \mathbf{u}'_\perp(\mathbf{x})$ is the velocity increment computed in planes perpendicular to the magnetic field, and $\mathbf{u}'_\perp$ is the two-component velocity fluctuation in such perpendicular planes. We show that the kinematics of the turbulence driven in our experiment is fully described with two lengthscales only: the injection scale l_i and the diffusion length associated to the injection scale $l_z^u(l_i)$. Indeed, $\langle \delta \mathbf{u'_\perp}^2 \rangle(\mathbf{r})$ computed for different operating conditions collapse onto a single plot, after normalizing the increment vector by the two aforementioned lengthcales. The second order structure function displays radically different shapes, whether turbulent scales lay below or above the $l_z^u(l_i)/h = 1$ threshold. More specifically, quasi-2D structures feature a very clear anisotropy caracterized by vertical iso-values of $\langle \delta \mathbf{u'_\perp}^2 \rangle$, which run parallel to the axis of the magnetic field. 3D structures, on the other hand, present circular iso-values of $\langle \delta \mathbf{u'_\perp}^2 \rangle$, in line with an isotropic distribution of kinetic energy.

The scale by scale flux of perpendicular turbulent kinetic energy along perpendicular scales is also evaluated experimentally, based on the third order structure function $\langle \|\delta \mathbf{u}'_\perp\|^2\, \delta \mathbf{u}'_\perp \rangle$. We show in particular that regardless of the dimensionality of the injection scale, there always exists a range of scales larger than the injection scale, which experiences an upscale flux of kinetic energy. This observation thus suggests that perpendicular kinetic energy always follows an inverse energy cascade, despite the scales supporting it being kinematically 3D.

References

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